AN APPROACH TO ECONOMIC ANALYSIS OF WATER RESOURCE SYSTEMS UNDER HIGH RAINFALL UNCERTAINTY[1]

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ABSTRACT - Most water supply systems are sized somewhat arbitrarily by either designing them to yield a firm supply for the worst drought of record or some smaller amount which way be decided by available resources. The first size estimate for most systems should be that capacity which maximizes expected net benefits, thus incorporating probability into the economic analysis. This is especially true for regions of high precipitation uncertainty. If, after that problem is solved, the client desires to choose another project size, the trade-offs are clearer. This paper will illustrate the principle by an example of sizing a small reservoir (tank) for the northeastern Brazilian semi-arid tropics (SAT).

Index terms: water management, optimization of water resources.

MÉTODO PARA ANÁLISE ECONÔMICA DE SISTEMAS PARA RECURSOS HÍDRICOS EM CONDIÇÕES DE ALTA INCERTEZA DE CHUVAS

RESUMO - A maioria dos sistemas de suprimento de água são dimensionados arbitrariamente pela sua projeção para assegurar um abastecimento garantido de água para a pior seca de um período escolhido ou de quantidades menores, que podem depender de recursos financeiros disponíveis. A primeira estimativa de tamanho do reservatório para a maioria dos sistemas poderá ter uma capacidade que maximize os lucros líquidos esperados, incorporando probabilidade na análise econômica. Isto é válido principalmente para regiões com alta incerteza de chuvas. Se, após a solução deste problema, o usuário quiser escolher outro projeto de tamanho diferente, as alternativas são claras. O presente trabalho ilustrará o princípio com uma análise do dimensionamento de de um pequeno servatório (barreiro) para o semi-árido da região Nordeste do Brasil.

Termos para indexação: manejo de água, otimização de recursos hídricos.

INTRODUCTION

In designing water resource systems, there are two separable problems that must be considered: One is what the design of the system should be; and second is what size to make the design. In systems that will be built over time, there is an additional question of when to build the various stages and what the sizes of the stages should be, sometimes called the "capacity-expansion problem".

In this paper, an approach to optimally size a samall water supply reservoir will be discussed, though the principles apply to flood control, hydropower, and other functions in the water resources sector. To illustrate the approach, we will assume the semi-arid tropics (SAT) climate that occurs in the Brazilian Northeast in which the annual precipitation occurs during a specific season (rainy season) with little or no precipitation during the dry season. Consequently, the amount of water stored for dry season irrigation (if any) is known before planting since dry season crops are usually planted after most of the rainy season has passed. However, available water for supplementary irrigation during the rainy season is not known. In these cases, some uncertainty is involved. Among the remaining simplifying assumptions, most of which are obvious, are: neglecting the timing of rainfall and the risk of sufficient rain not occuring during the previous year.

Unlike the SAT in India, the SAT surrounding Petrolina, Brazil, does not have sufficient rain during its rainy season to utilize small reservoirs (tanks) for off-season crops. The average annual precipitation is about 380 mm. Also, the time that rain comes during the rainy season is highly variable. This is unlike the SAT of India in which a rainy (monsoon) season delivers more water and is more predictable. In the northeastern Brazilian

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SAT, the precipitation may be spread over the whole season or occur in concentrated storms. Normally, there is not enough precipitation to irrigate from tanks during the dry season. The purpose of tanks in this paper, then, will be for supplemental irrigation during the rainy season, which interestingly enough, occurs during the summer months.

Small reservoirs, called "tanks" in India, are either constructed by small earth dams, two to three meters high situated across a gulley draining a subcatchment, or they are dug ponds located in a sloping area to trap water (Sharma 1981). Normally, the design and size of these traditional water harvesting devices is not optimized. The efficiency of many, if not most, tank systems could be increased two to five fold by proper design, sizing, and operation (Sharma & Helweg 1983).

In common practice, risk is defined as future events of which the probability distribution is known and uncertainty is defined as future events of which the probability distribution is not known. Practically speaking, future events may lie along a continuum between pure risk and pure uncertainty. The precipitation in both the SAT of India and Brazil is uncertain, but it is more uncertain in Brazil; nevertheless, we may assign a probability distribution to it as long as we remember that it is very approximate. Because of the greater precipitation uncertainty in Brazil, the potential gains from including probability in the economic analysis are greater.

ECONOMIC PRINCIPLES INVOLVED

Unless formal multi-objective analysis is contemplated, the first attempt to optimally size a water resources project should maximize net benefits. If the decision maker desires a different size, he at least knows the monetary trade-offs. That is, he will know how much it is costing to depart from the most economic size.

Four items of data are required for an economic analysis: benefits, costs, discount rate, and project life. In this paper, we will assume all benefits and costs have been discounted to present worth. Moreover, we will assume the costs are least costs. That is, the small reservoir (tank) which represents the costs of water, is optimally designed. The design problem is nontrivial and has been treated in some depth by Sharma (1981), Sharma & Helweg (1982) and Helweg & Sharma (1983).

For a small reservoir, the benefits are the increased income from utilizing the reservoir. The costs are the construction and operation and maintenance (O & M) costs of the reservoir. As usual, the costs are relatively straightforward, but calculating the benefits has several pitfalls.

First, in order to find benefits, we must have a production function for the crop in the form:

$$Y = f(Q, K) \quad (1)$$

in which Y is the crop yield in metric tons/ha, Q is the amount of water required in meters and K is all other factors such as soil, farming techniques, etc. Assuming all other factors remain constant and neglecting the timing of irrigation effects, we may approximate the production function as a quadratic equation,

$$Y = f(Q) = a_1 + a_2 Q + a_3 Q^2 \quad (2)$$

in which a's are coefficients found by regression analysis. Fig. 1 shows such a production function for oranges in California.

Once the production function is determined, the benefit becomes

$$NPW = P f(Q) - C_1 Q - C_2, \quad (3)$$

in which NPW is the present worth of net benefits P, is the price of crop yield, C_1 is the cost of water (cost of reservoir construction and its operations and maintenance cost) and C_2 is all other costs (e.g. cost of production, irrigation labour cost etc.). The optimum reservoir size is when the reservoir volume, V^*, plus precipitation delivers the amount of water that maximizes NPW, or:

$$\frac{d(NPW)}{dQ} = P f'(Q) - C_1 = 0 \quad (4)$$

The problem is that Q is not known, so P f(Q) may be estimated statistically by its expected value, EV.

DETERMINING THE EXPECTED VALUE

In order to determine expected value, we must assume

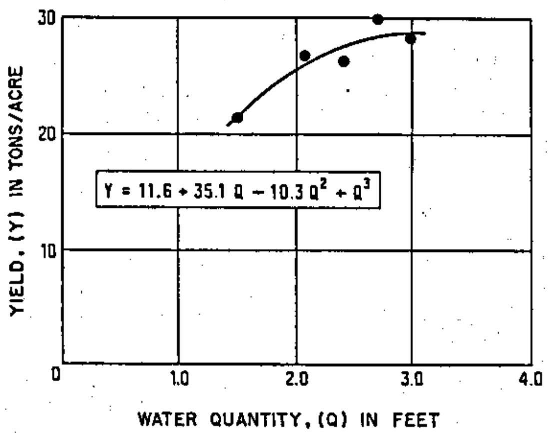

FIG. 1. Production function for citrus, yield versus water quantity.

a cumulative distribution function (CDF) for precipitation (recall that that the CDF is the integral of the probability distribution function PDF). Even though precipitation is uncertain, the historic record, hopefully longer than 20 years, may be approximated by a log-normal distribution. In this paper, we will use the normal distribution without loss of generality to simplify the example.

The expected value, EV, then may be defined as:

$$EV = \sum_j P_j \sum_i \Delta p_i f_j (Q_i) \quad (5)$$

in which p_i is the probability of having Q_i amount of water available for the crop, P_j is the price of crop j. Note we use capital "P" for price and lower case "p" for probability, Δp_i being the interval between probabilities. Equation (5) is illustrated by Fig. 2 which shows how the produc-

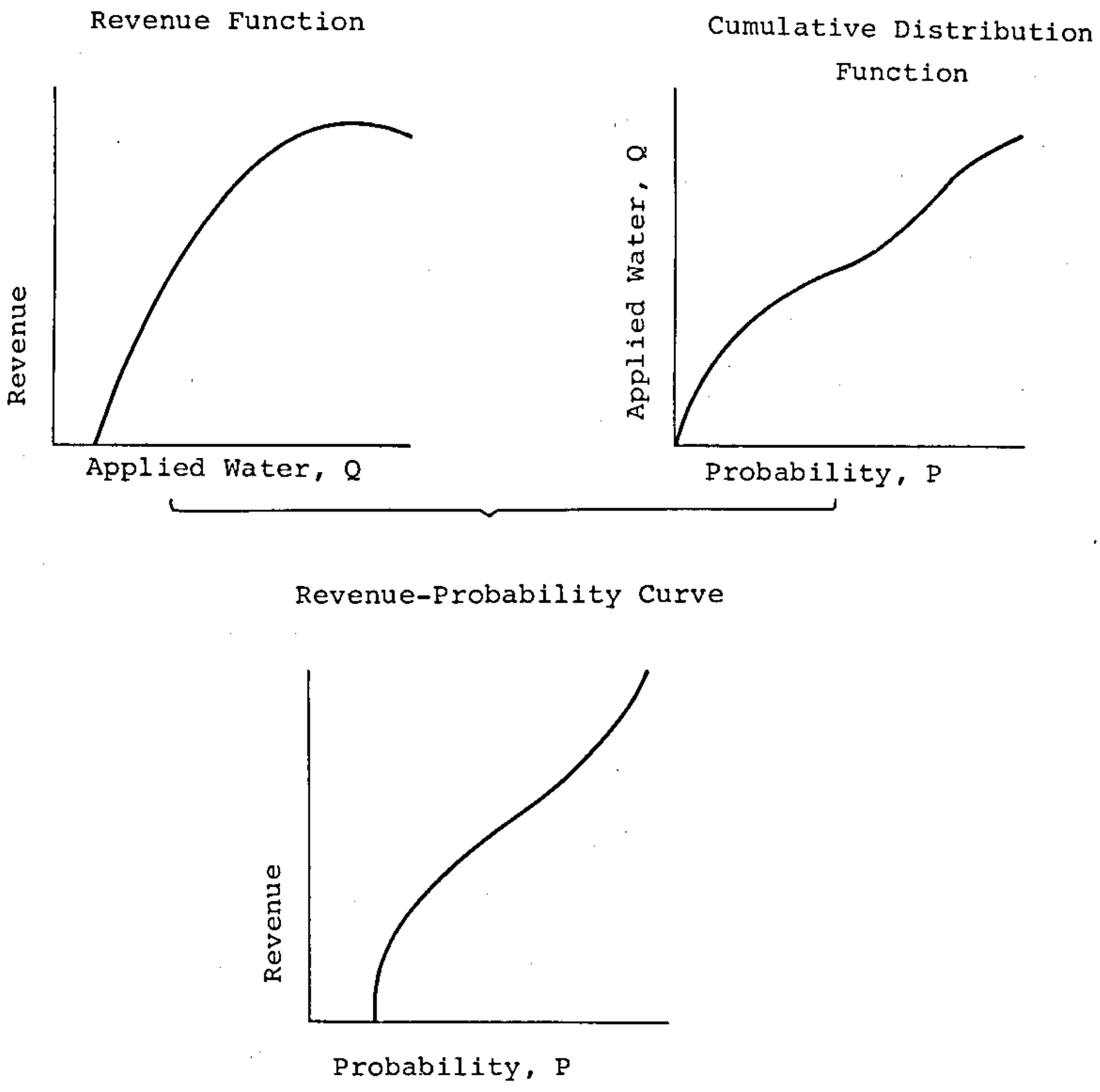

FIG. 2. Deriving a revenue-probability curve from the revenue function and cumulative distribution function of water water available for crop production.

tion function (converted to a revenue function by multiplying production Y price P) is combined with a CDF to produce a revenue-probability curve.

For example, assume a CDF of the water availability to crop which resembles the precipitation of the Brazilian SAT given in Table 1 and a production function of a crop given in Table 2. Table 3 solves equation (5) for one crop. Notice the average water requirement from Table 1 is

$$\text{Ave. Precip.} = \Sigma\, \Delta p_i\, Q_i \quad (6)$$

which equals 0.448 m.

FINDING THE OPTIMAL RESERVOIR VOLUME, V*

Recall that the optimal reservoir volume, V*, will be that volume which yields an optimum amount of water available to the crop Q*, to solve equation (4), ie.:

$$Q^* = V^* \left(1 - \sum_{i^*=1}^{n} \Delta\, p_i\right) + \sum_{i^*=1}^{n} \Delta\, p_i\, f(Q_i) \quad (7)$$

in which i* is the probability at which $V^* = Q_i$. To find V* the expected benefits without the tank must be subtracted from the benefits with the tank. This is the "with--without" principle in economic analysis.

Using the costs in Table 4 and assuming reservoir costs in column (7) of Table 5, Table 5 solves equation (4) by finding the maximum NPW directly rather than analytically. Fig. 3 is a graphical solution of Table 5.

TABLE 1. Probability of receiving various amounts of water for crop.

Q_1 *	p_i **	Δp_i
0.0	0.00	
0.1	0.02	0.10
0.2	0.10	
0.3	0.25	0.30
0.4	0.40	0.20
0.5	0.70	0.20
0.6	0.80	
0.7	0.85	0.10
0.8	0.90	
0.9	0.92	0.06
1.0	0.96	
1.1	0.99	0.04
1.2	1.00	

* Q_i (Water use) = precipitation + irrigation - appropriate losses

** p_i = the probability of getting at least Q_i

TABLE 2. Crop production function*.

Q_i (meters)	Y (metric tons/ha)
0.1	0
0.2	0
0.3	17
0.4	30
0.5	40
0.6	52
0.7	65
0.8	72
0.9	80
1.0	82
1.1	60

* Assumed values for illustration.

TABLE 3. Finding the expected yield, Ey, of the crop.

(1) Q_i	(2) $f(Q_i)$	(3) Δp_i	(4) $\Delta p_i f(Q_i)$ (2) x (3)	(5) EY Σ (4)
0.1	0	0.10	0.0	0.0
0.3	17	0.30	5.1	5.1
0.4	30	0.20	6.0	11.1
0.5	40	0.20	8.0	19.1
0.7	65	0.10	6.5	25.6
0.9	80	0.06	4.8	30.4
1.1	80	0.04	3.2	33.6

The expected yield of this crop is 33.6 tons/ha.

CONCLUSIONS

This analysis illustrates three errors to avoid in designing a small reservoir:

1. Not to design the reservoir for average runoff or expected runoff. This would yield a design related to a precipitation of 0.448 m and clearly give negative net benefits.

2. Not to design the reservoir to fit the maximum expected value of the crop. This would design a reservoir to yield 1 m of water, again yielding negative net benefits (NPW < 0).

3. Not to omit the "with-without" principle. Doing this overestimates the project benefits.

TABLE 4. Citrus orchard production cost for San Diego County, California, USA.

I. Fixed costs/ha		
Operating		
Irrigation Labor*	$133	
Pest and Disease Control	272	
Fertilization	104	
Weed Control	148	
Pruning & Brush Disposal	49	
Forest Protection	136	
Misc., Tree Core Replace, etc.	49	
	$891	
Cash overhead		
Taxes	$297	
Maintenance & Repair	86	
General expenses	99	
Management	148	
	$630	
Investment		
Depreciation	$662	
Interest on investment	1085	
	$1747	$3268/ha
II. Variable cost		$ 9.40/ton
Harvesting labor		
III. Benefit		
Revenue from crop Sales		$102/ton**

* This is assumed to be fixed though in actuality it varies slightly with the amount of applied irrigation water (Q).

** This price is not part of the study.

TABLE 5. Finding the optimal size of a small reservoir (tank).

(1) V	(2) EY	(3) B_w* 102 (2)	(4) B_{wo}* 102 x 33.6	(5) B (3) - (4)	(6) C_1	(7) C_2	(8) B - C (5) - (6) - (7)
0.0	33.6	3427	3427	0	120	3268	-3388
0.2	33.6	3427	3427	0	150	3268	-3418
0.4	40.5	4131	3427	704	200	3550	-3046
0.6	64.1	6538	3427	3111	300	3748	- 917
0.8	79.3	8089	3427	4662	500	3945	217 (max)
1.0	82.0	8364	3427	4937	1000	4039	- 102
1.2	82.0	8364	3427	4937	1800	4039	- 902

* B_W and B_{WO} are benefits with and without the reservoir, respectively.

Note: The Ey does not change from a V of 1 to 1.2 because one would not overirrigate so as to decrease crop yield.

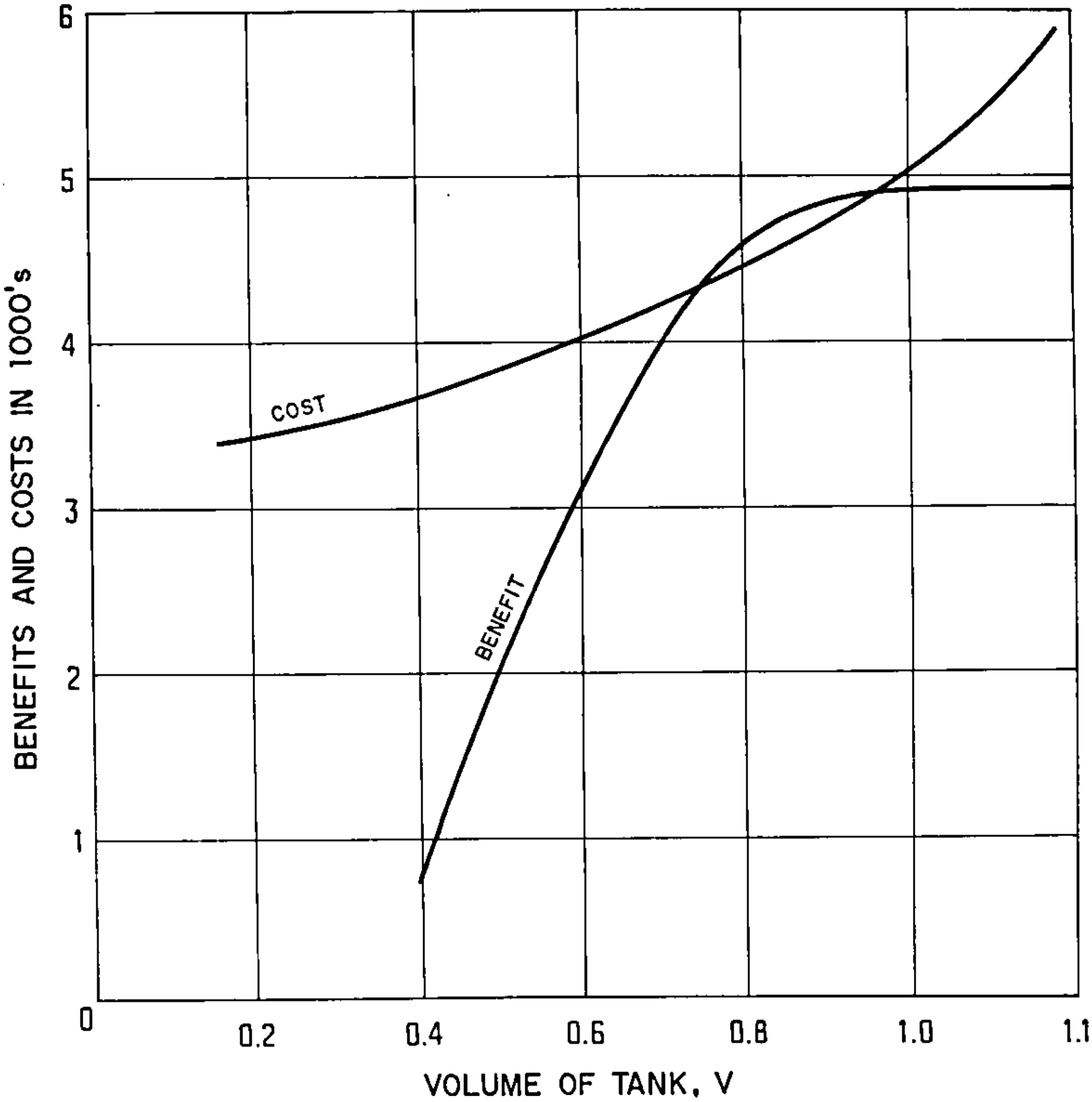

FIG. 3. Graph of Table 5 showing benefits and costs vs. Tank size.

Most of the numbers in this illustration have been arbitrarily chosen to illustrate the procedure. Before applying this method to an actual project, obtain the correct production functions, precipitation, CDF, etc. A more rigorous approach would use conditional probability including the risk of not receiving rain the previous year. Finally this paper has clearly demonstrated the approach that should be taken to optimally design of small water resource systems for areas of high rainfall uncertainty e g the Brazilian Northeast.

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